

Approximately Optimal Testing Strategy and Surveillance Test Interval for a 2-out-of-3 Standby System

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Abstract

A cost-effective maintenance schedule, including testing strategy and surveillance test interval, for a 2-out-of-3 standby system is presented. For systems with identical units, uniformly staggered testing is shown to be the best testing strategy. Moreover, the optimal test interval can be approximated by that of the comparable two-unit parallel systems, provided that all units are identical.

1 Introduction

Engineering safety systems are usually standby systems whose failures are hidden and can be discovered only by inspection or at the next activation. Earlier research generally aims to find the optimal preventive maintenance schedule that minimizes average system unavailability.

Apostolakis and Chu (1980) derive analytical equations for the average system unavailability of m -out-of- n systems (for $n \leq 3$) under both (almost) simultaneous and uniformly staggered testing. Vesely and Goldberg (1977) developed a computer code—FRANTIC—to compute time-dependent system unavailability according to the time-dependent unavailability of the system's components. Vaurio and Sciaudone (1979) derive analytical expressions for the average system unavailability based on both staggered and (almost) simultaneous testing schemes. They also discuss the average unavailability of m -out-of- n systems (for $n \leq 4$). The optimal solution is obtained based on the Newton-Raphson method incorporated in a computer code, ICARUS.

However, the optimal testing schedule minimizing system unavailability may result in frequent tests or maintenance, increasing not only the test or maintenance cost, but also the probability of failure due to imperfect maintenance. In this study, we provide a cost-effective model to obtain an approximately optimal preventive maintenance schedule for 2-out-of-3 standby systems, taking into account both system unavailability and also the maintenance costs.

2 Assumption and Objective Function

For systems with identical units, it is reasonable to test all units at the same test interval. Figure 1 shows the testing schedule of a 2-out-of-3 system using equal test intervals. Without loss of generality, assume that the i^{th} cycle begins with the i^{th} test of unit 1 at time O_i . After testing, unit 1 is restored at time A_i , and stays in the standby status until the next test at O_{i+1} . Similarly, unit 2 is tested at B_i and restored at C_i , and unit 3 is tested at D_i and restored at E_i , respectively. The cycle ends just before the $(i+1)^{\text{st}}$ test of unit 1; that is, at point O_{i+1} .

The time difference between the tests of units j and k is assumed to be constant for all cycle i , and denoted by L_{jk} . The expected downtime for testing, restorative maintenance, and repair for each unit is denoted by τ . The decision variables in this model are surveillance test interval (i.e., STI), and the time lags between the tests of units (i.e., L_{12} and L_{23}). The assumptions for this study are described in Section 2.1, and the objective function, taking both system unavailability and maintenance cost into account, is presented in Section 2.2.

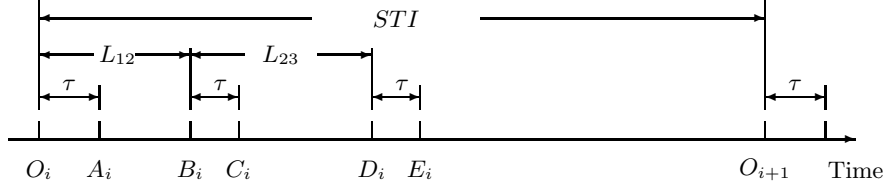


Figure 1: General Maintenance Schedule for a 2-out-of-3 System

2.1 Assumptions

1. All units are tested at regular, fixed interval, denoted by STI .
2. A unit is either operable or failed. There are no partially degraded states.
3. Units can fail either because of random failure during standby (with constant failure rate λ), or because of on demand (with failure probability ρ).
4. Following surveillance testing, some minimal level of restorative maintenance is performed if the unit is found to be in the operable state during the test; otherwise, the unit is fully repaired or replaced.
5. A unit is unavailable during testing, restorative maintenance, and repair.
6. A unit is as good as new immediately after its restoration after maintenance or repair.
7. Units are assumed to fail independently of each other.

2.2 Objective Function

In general, a 2-out-of-3 system has the same physical structure as a three-unit parallel system, as shown in Figure 2. However, unlike three-unit parallel systems, (which are available if any one unit is available), 2-out-of-3 systems are operable only if at least two units are available. Such systems can also be represented as shown in Figure 3, with three two-unit parallel subsystems in series. Using the rare event approximation, the system unavailability for a 2-out-of-3 system when all units are on standby status can be approximated as

$$u(t) \approx u_1(x_1)u_2(x_2) + u_1(x_1)u_3(x_3) + u_2(x_2)u_3(x_3)$$

where x_j is the age of unit j at time t , and $u_j(x_j)$ is the time-dependent unavailability of unit j at age x_j . However, when one of units is being tested, the system becomes a two-out-of-two systems and the system unavailability can also be approximated as

$$u(t) \approx u_j(x_j) + u_k(x_k)$$

where units j and k are the units not being tested. Moreover, the time-dependent unavailability of unit j at age x_j is given by

$$u_j(x_j) \approx \begin{cases} \rho + (1 - \rho)\lambda x_j & \text{if unit } j \text{ is on standby, i.e., } 0 \leq x_j < STI - \tau \\ 1 & \text{if unit } j \text{ is being tested, i.e., } STI - \tau \leq x_j \leq STI \end{cases}$$

The key parameters of cost are: the cost per cycle for testing and restorative maintenance each unit, C_T ; and the additional cost incurred if repair or replacement is needed, C_F . Therefore, the expected maintenance cost per cycle is given by

$$C(STI) = C_T + C_F u(STI - \tau).$$

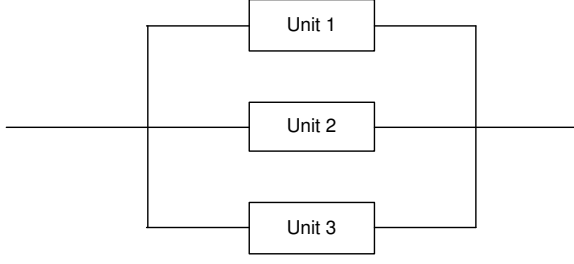


Figure 2: A 2-out-of-3 System

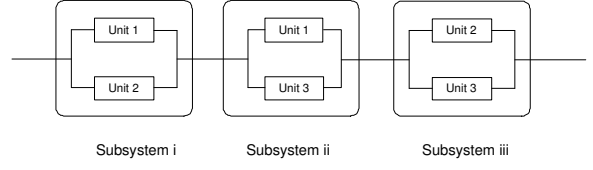


Figure 3: A Functionally Equivalent Representation of a 2-out-of-3 System

Then, computing the maintenance cost and expected downtime for each period and dividing by the surveillance test interval yields the objective function as

$$M(STI, L_{12}, L_{23}) = C_u \bar{U}(STI, L) + \frac{3}{STI} [C_T + C_F u(STI)]$$

where C_u is the expected loss per time unit of unavailability and $\bar{U}(STI, L_{12}, L_{23})$ is the average system unavailability per cycle that depends on both the test interval of the system and the values of time lags between the tests of units. The expected loss per time unit of unavailability, C_u , is the average cost that managers would expect to incur if the system in question were unavailable for one time unit (taking into account both the likelihood that the system would be required during that time, and the consequences if it were unavailable at the time of a demand). It is clear that the objective function is a weighted sum of average system unavailability and average maintenance cost.

3 Optimal Testing Schedule

3.1 Testing Strategy

For any given test interval STI , if either of the optimal time lags L_{12}^* or L_{23}^* is not at the edge of the feasible region, then it must satisfy

$$\begin{aligned} \frac{\partial}{\partial L_{12}} M(STI, L_{12}, L_{23})|_{(L_{12}^*, L_{23}^*)} &= 0 \quad \text{or} \\ \frac{\partial}{\partial L_{23}} M(STI, L_{12}, L_{23})|_{(L_{12}^*, L_{23}^*)} &= 0, \end{aligned} \quad (1)$$

respectively. The solution satisfies Eq (1) is

$$\begin{cases} L_{12}^* = \frac{1}{3} STI \\ L_{23}^* = \frac{1}{3} STI \end{cases}$$

Since the objective function is convex with respect to L_{12} and L_{23} , for any given value of STI . That is, the time lags $L_{12} = L_{23} = \frac{1}{3} STI$ minimize the objective function if the test interval STI is pre-specified. In other words, the optimal testing strategy for a 2-out-of-3 system with identical units is uniformly staggered testing.

3.2 Surveillance Test Interval

As shown in Figure 3, a 2-out-of-3 system can also be represented as three two-unit parallel subsystems in series. Since three units are identical, three subsystems are also identical. Wang (2004) shows that the optimal test interval for each unit in a series system is quite close to the optimal test interval that would have been obtained if that unit had been considered alone as a single-unit system. Since three two-parallel subsystems shown in Figure 3 are identical, the optimal test interval of the system should be close to that of any subsystem. Therefore, it is reasonable to hypothesize that the optimal test interval for a 2-out-of-3 system would be close to that of the two-parallel subsystem provided that all units are identical.

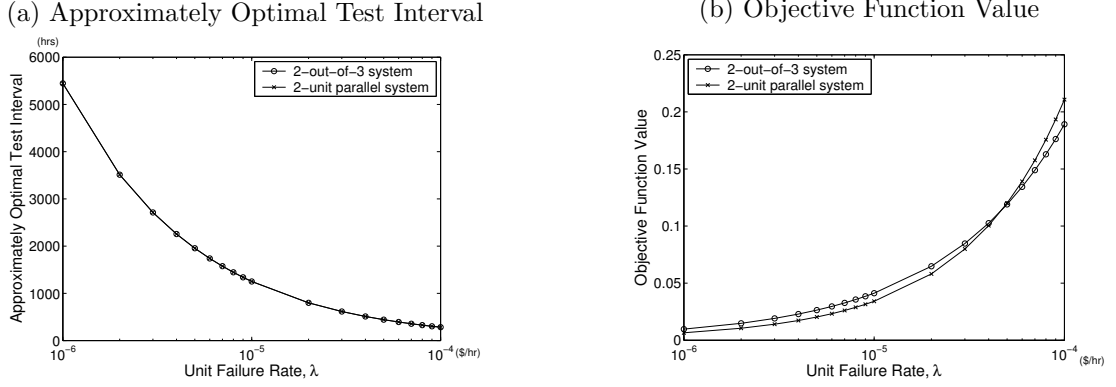


Figure 4: 2-out-of-3 Systems vs. Two-Unit Parallel Systems with Identical Units (for the case in which $C_u = \$100/\text{hr}$, $\tau = 1 \text{ hr}$, $\rho = 10^{-3}/\text{demand}$, $C_T = \$10/\text{test}$, $C_F = \$100/\text{repair}$)

Figure 4 compares the optimal test interval and the objective function for a 2-out-of-3 system with those for a two-unit parallel system with identical units. Figure 4(a) shows that there is no significant difference in the approximately optimal test interval for these two systems, even though the objective function values are different, as shown in Figure 4(b). Therefore, the approximately optimal test interval for a two-unit parallel system that is also discussed in Wang (2004) can also be used for a 2-out-of-3 system with identical units.

4 Conclusion

An approximately optimal testing schedule, taking into account both the system unavailability and maintenance cost, for 2-out-of-3 systems with identical units was obtained. Uniformly staggered testing was found to be approximately optimal for 2-out-of-3 systems with identical units. Based on uniformly staggered testing, we discovered that the approximately optimal test interval of a 2-out-of-3 system can be approximated by that of the comparable two-unit parallel system, provided that all units are identical. This motivates us to hypothesize that the approximately optimal test interval of a $(N - 1) - \text{out} - \text{of} - N$ system may perhaps be approximated by that of the comparable two-unit parallel system, provided that all units are identical. This hypothesis would be investigated in future work.

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